

50010-190

Chmutov's Polynomial

Towards a Symmetric Tutte Polynomial

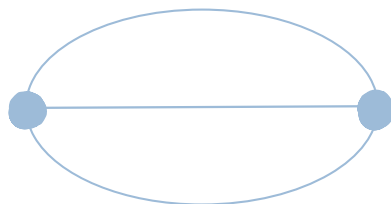
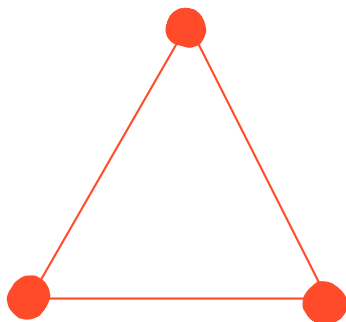
Chmutov's First Formulation

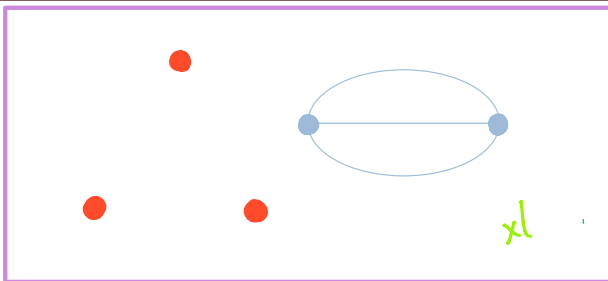
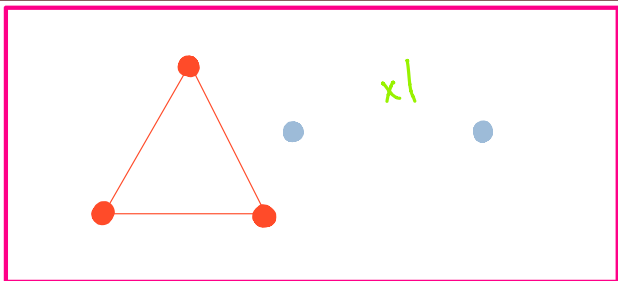
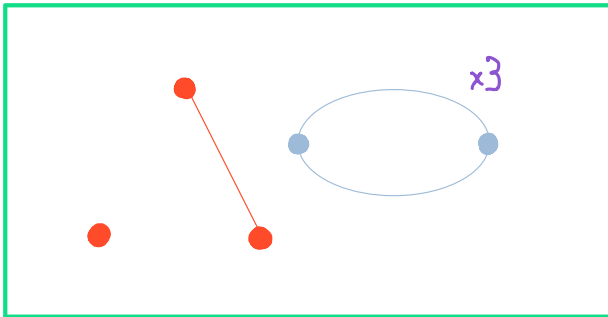
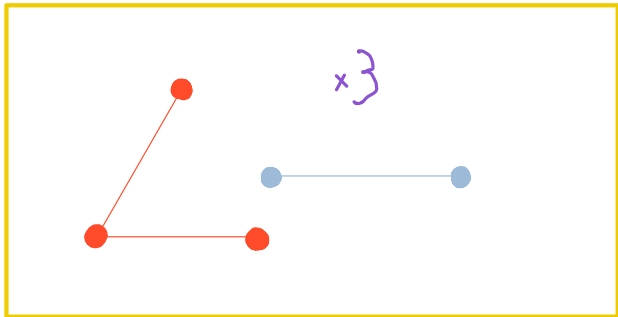
Select each edge to either be in the graph, or not in the graph but in its dual.

Assign to each connected component K a label p_k in the original graph or q_k in the dual graph, where k is the number of vertices in K .

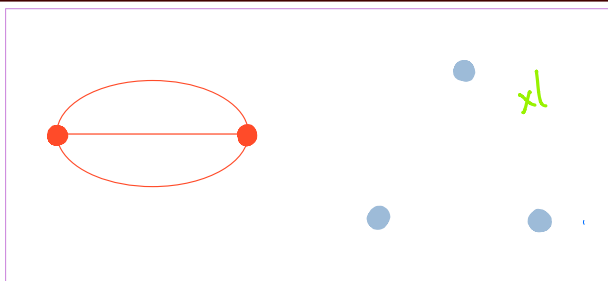
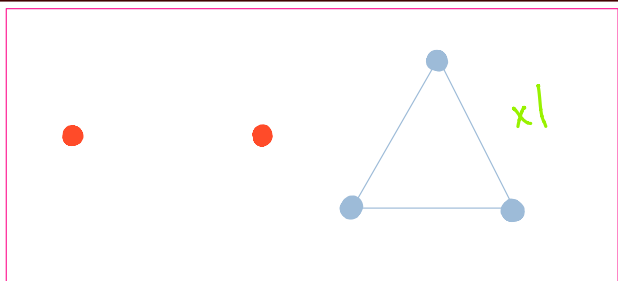
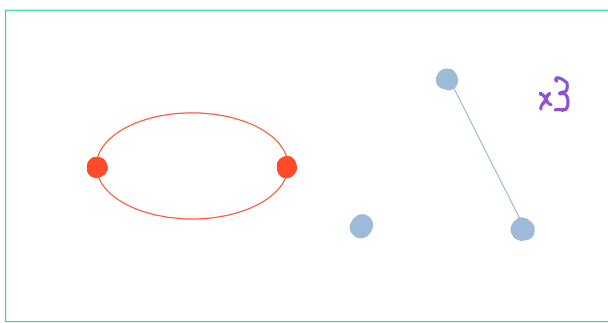
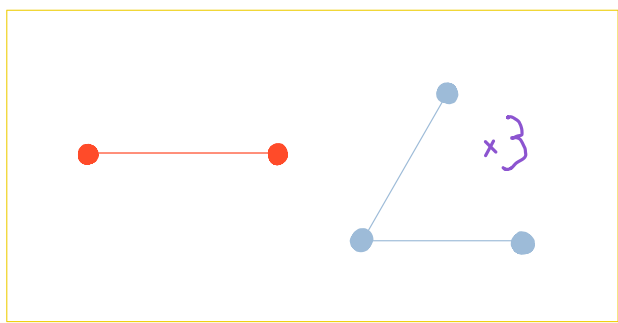
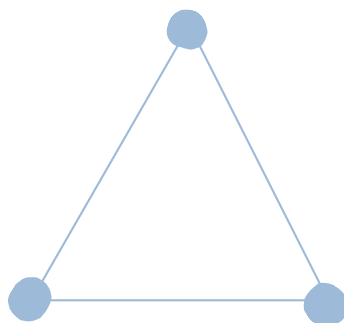
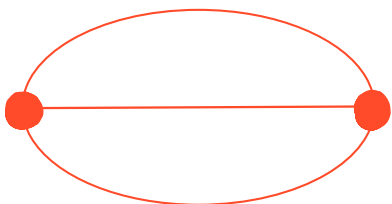
Take the product of these labels, and sum over each state.

$$\sum_{\sigma \subseteq E} \left(\prod_{K \in \sigma} p_{|V(K)|} \prod_{K \in \sigma^*} q_{|V(K^*)|} \right)$$





$$p_3 q_1^2 + 3 p_3 q_2 + 3 p_1 p_2 q_2 + p_1^3 q_2$$



$p_1^2 q_3 + 3p_2 q_3 + 3p_2 q_1 q_2 + p_2 q_1^3$
$p_3 q_1^2 + 3p_3 q_2 + 3p_1 p_2 q_2 + p_1^3 q_2$

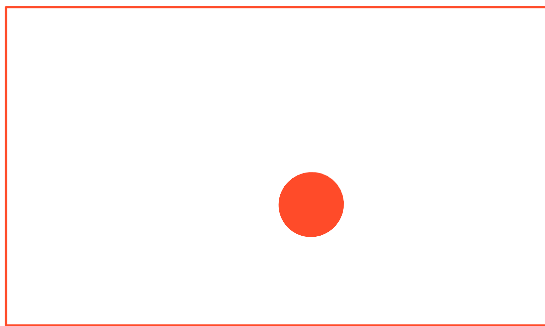
It's Dual Reversing!

... Define it to be symmetric.

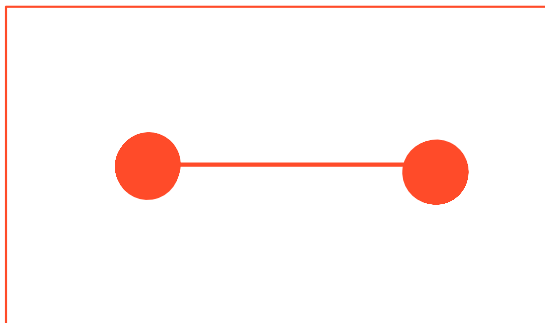
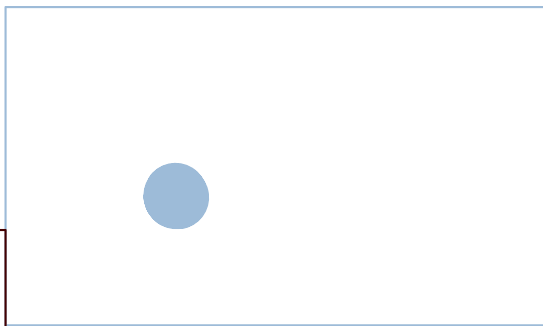
p over x

q over y

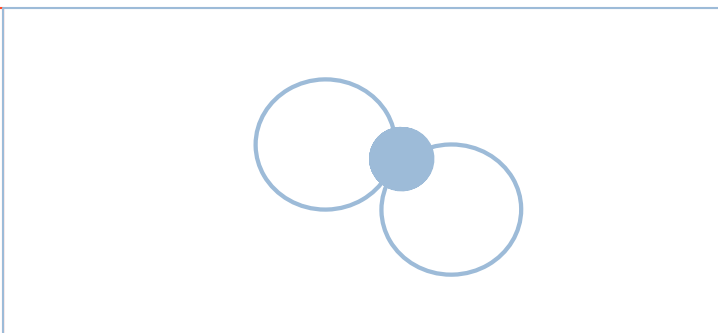
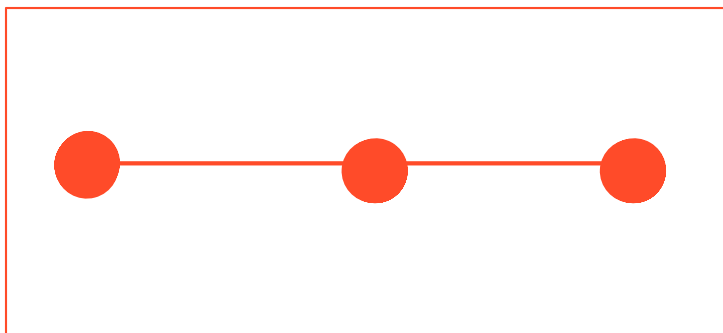
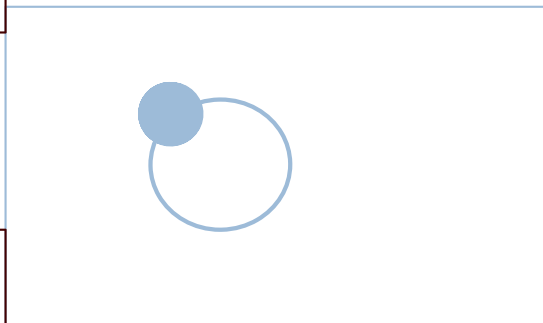
Exercises & Examples



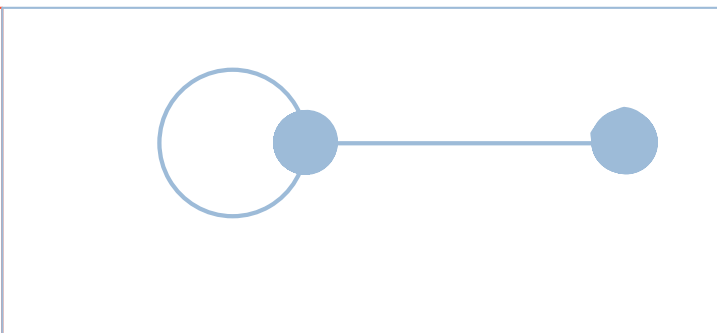
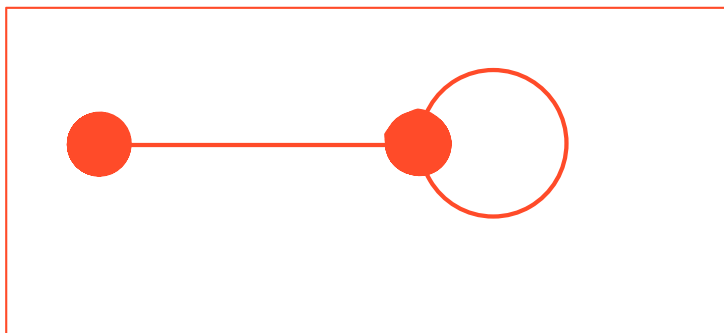
$$P_1 a_1$$



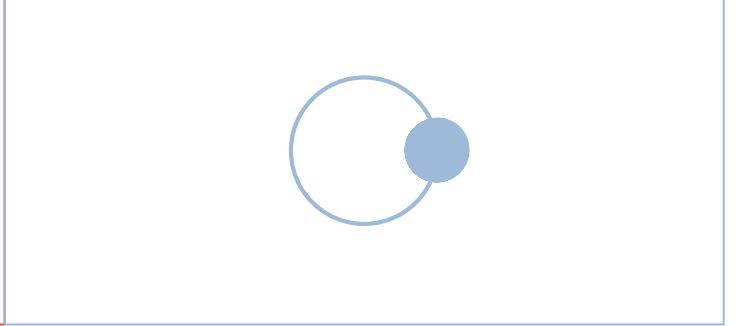
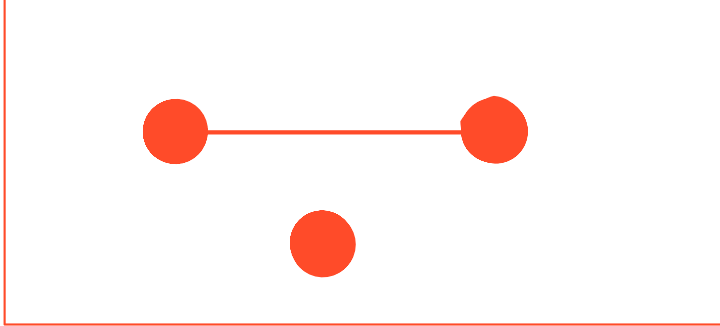
$$P_2 a_1 + P_1^2 a_1$$



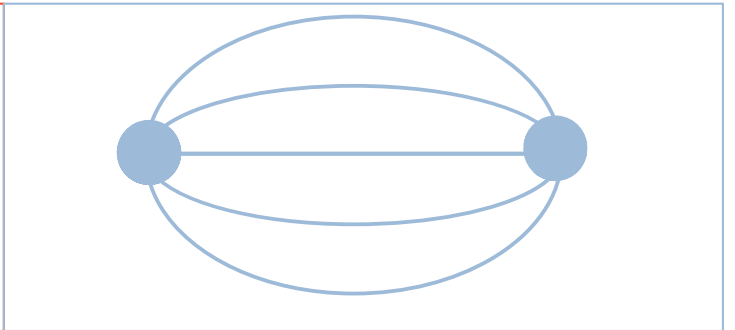
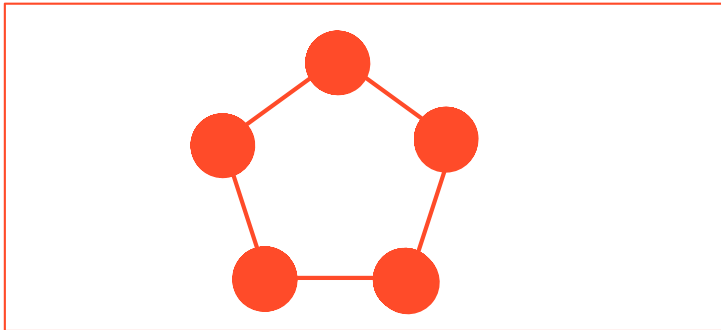
$$P_3 a_1 + 2P_1 P_2 a_1 + P_1^3 a_1$$



$$P_2 a_1^2 + P_2 a_2 + P_1^2 a_1^2 + P_1^2 a_2$$

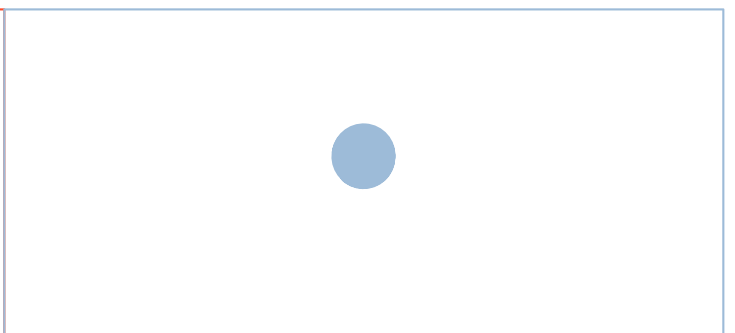
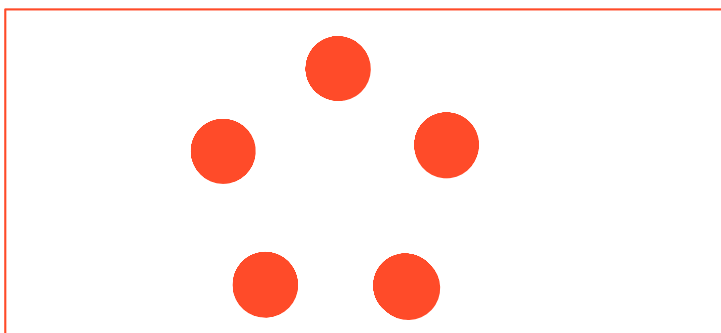


$$P_1 P_2 a_1 + P_1^3 a_1$$

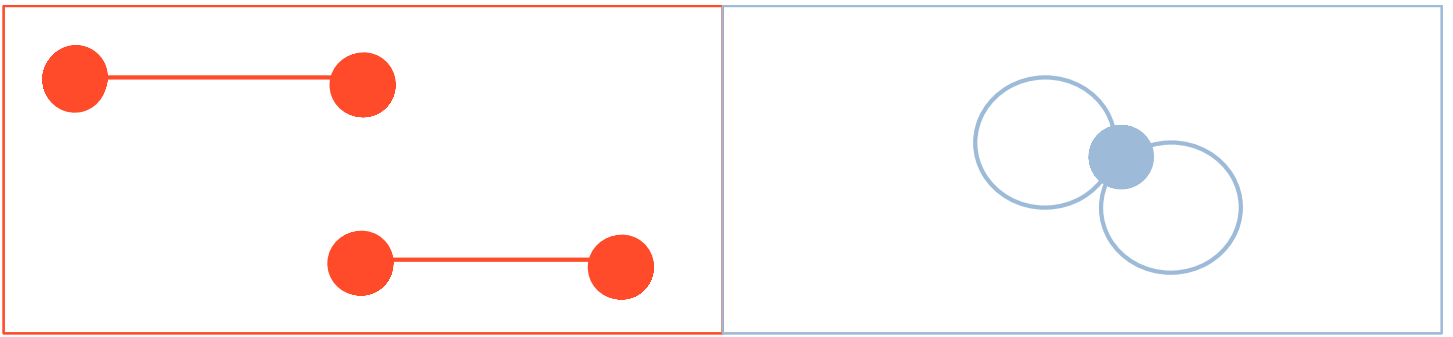


$$P_5 a_1^2 + S P_5 a_2 + S P_4 P_1 a_2 + S P_3 P_2 a_2$$

$$+ S P_3 P_1^2 a_2 + S P_2^2 P_1 a_2 + S P_2 P_1^3 a_2 + P_1^5 a_2$$



$$P_1^5 a_1$$

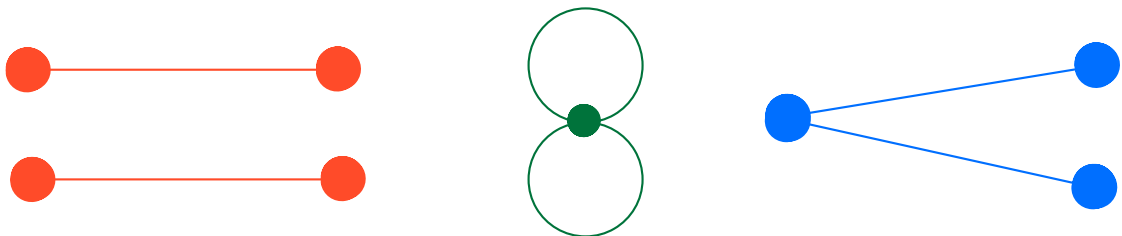


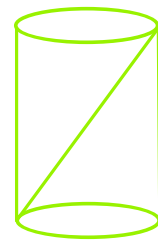
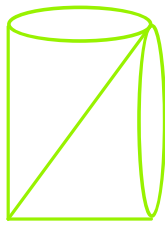
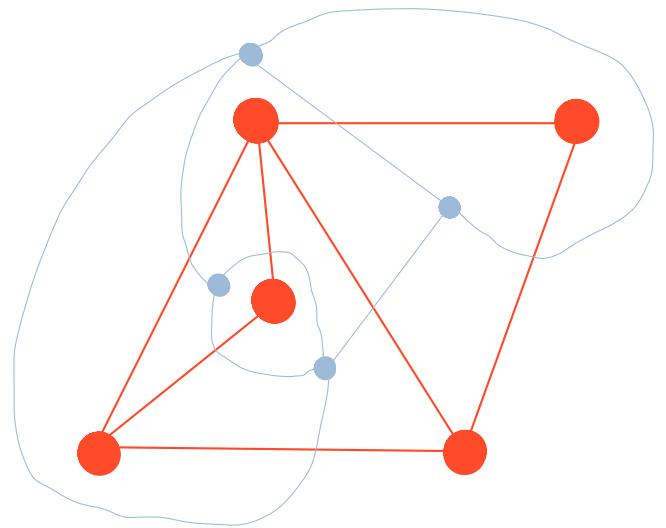
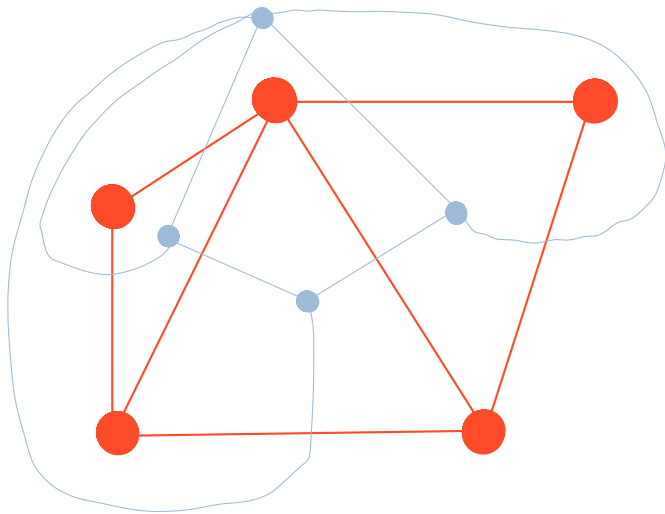
$$p_2^2 a_1 + 2p_1 p_2 a_1 + p_1^2 a_1$$



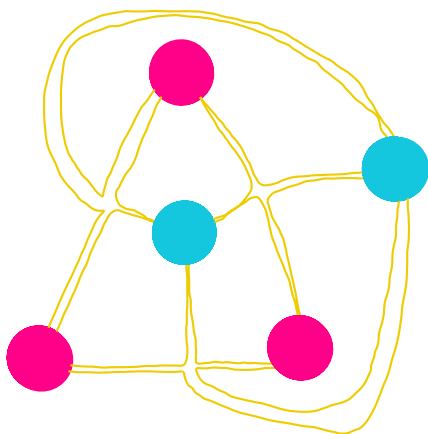
$$p_1 a_5$$

Huh

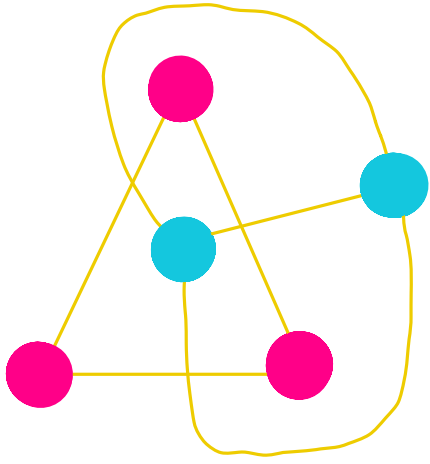




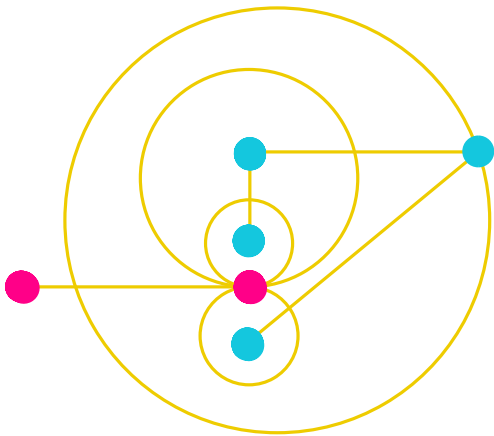
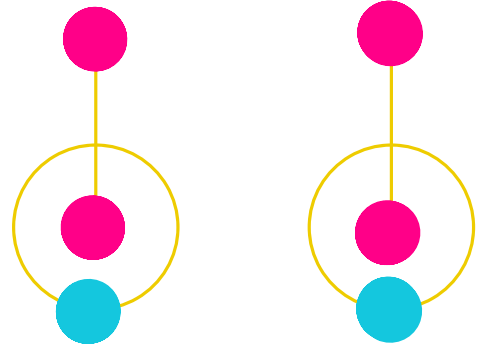
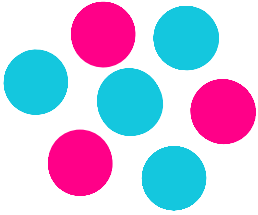
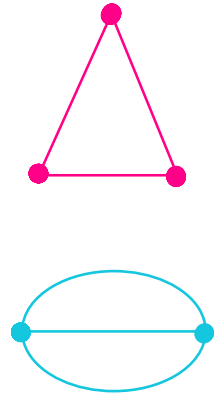
I Introducing...
Sesquigraphs

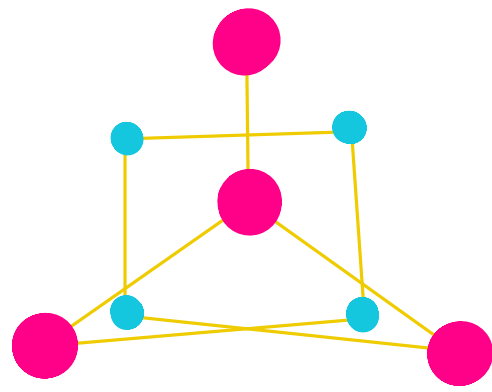
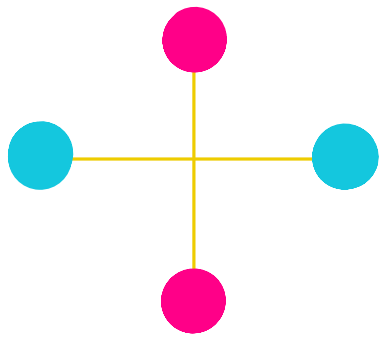


$$S = \langle A, B, E \rangle$$



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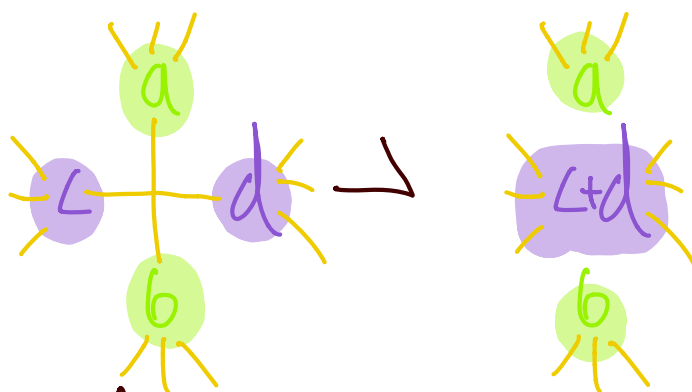




Deletion Contraction and the Second Formulation

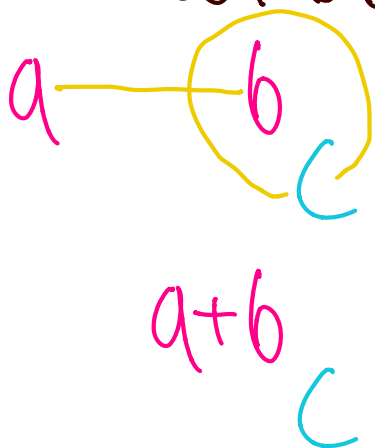
Weighting: Associate with each element in A and B a natural number, its weight, defaulting to one.

Deletion
Contraction

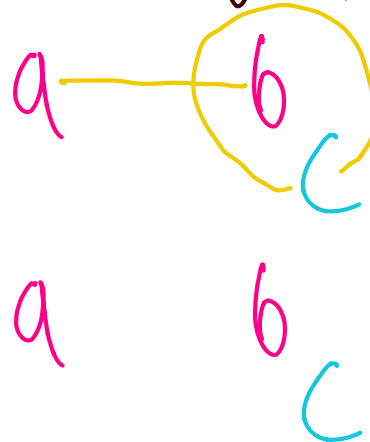


is a contraction on the violet set and a deletion on the line set.

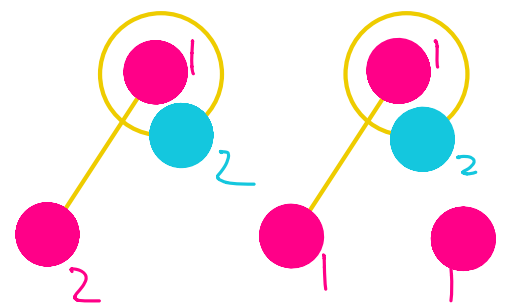
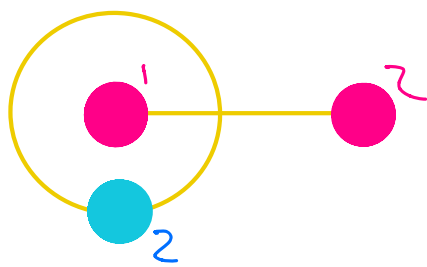
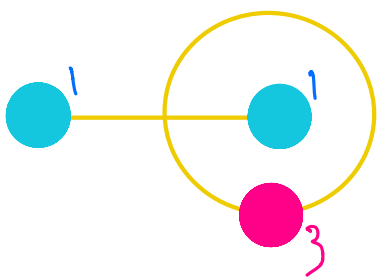
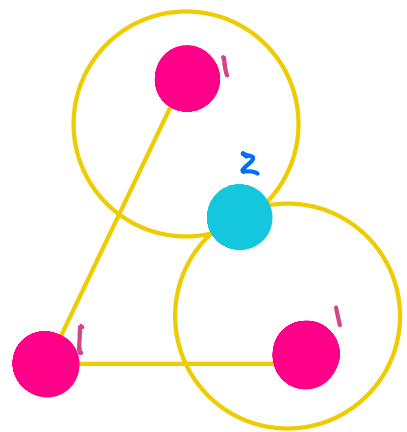
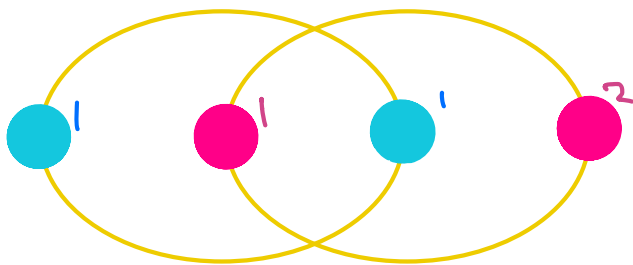
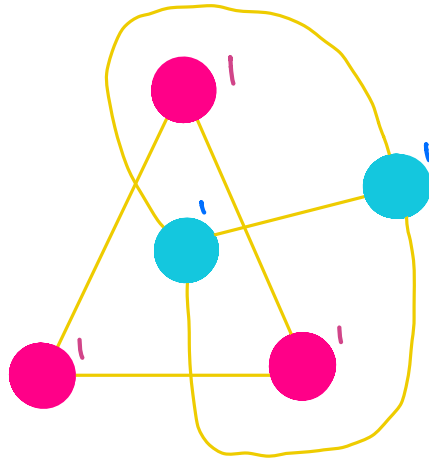
Isthmus Contraction



Isthmus Deletion



$$D(S) = D(S/e) + D(S-e)$$



311 32 32 122 32 122 122 1112

$$p_3 a_1^2 + 3p_3 a_2 + 3p_1 p_2 a_2 + p_1^3 a_2$$

Stanley:

$$\chi[P_n] = (-1)^{n-1} D[P_n, -1]$$

Dichromatic:

$$Z[a, b] = b^{n-1} D\left[\frac{a}{b}, b\right]$$